Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Winter Examination-2015

Subject Name: Mathematical Methods-I

Subject Code: 5SC03MAC1 **Branch: M. Sc. (Mathematics)**

Semester: 3 Date: 03/12/2015 Time: 02:30 To 05:30 Marks: 70

Instructions:

(1) Use of Programmable calculator and any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

SECTION – I

Q-1 **Attempt the Following questions** **(07)**

a. Check whether given function is even or odd.

$$f(x) = \begin{cases} -a\,; & -c < x < 0 \\ a\,; & 0 < x < c \end{cases}$$

b. In the Fourier series expansion of $f(x) = x^3$ in $(-\pi, \pi)$, the value of a_n is ……

c. State Convolution theorem for Fourier transform.

d. Find $1 * 1$.

e. State Parseval's identity for Fourier transform.

f. Fourier transform is linear. Determine whether statement is True or False.

g. If $f(t)$ is a periodic function with period T, then $L[f(t)] = \int_0^T e^{-st} \cdot f(t)\, dt$. Determine whether statement is True or False.

Q-2 **Attempt all questions** **(14)**

a. Find the Fourier series with period 2 to represent $f(x) = x^2 + x$ in the interval $-1 < x < 1$. **(07)**

b. Find the Fourier transform of $f(x) = e^{-\frac{x^2}{2}}$. **(05)**

	c.	If $F(s)$ is the Fourier transform of $f(x)$, then prove that the Fourier transform of $f(ax)$ is $\frac{1}{a} F\left(\frac{s}{a}\right)$.	**(02)**

OR

Q-2		**Attempt all questions**	**(14)**
	a.	Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $0 \le x \le 2\pi$.	**(07)**
	b.	Find the Fourier transform of $(x) = e^{-a\|x\|}, -\infty < x < \infty$.	**(05)**
	c.	Prove that $F\{f(x) \cos ax\} = \frac{1}{2}[F(s+a) + F(s-a)]$.	**(02)**
Q-3		**Attempt all questions**	**(14)**
	a.	Find the half range Fourier sine series of $f(x) = \frac{\pi}{2} - x \text{in} 0 < x < \pi$.	**(05)**
	b.	Find the Fourier sine transform of $f(x) = \begin{cases} 0, & 0 < x < a \\ x, & a \le x \le b \\ 0, & x > b \end{cases}$.	**(05)**
	c.	Find the Laplace transform of $i) \sin^3 2t$, $ii) \frac{1-\cos t}{t}$.	**(04)**

OR

Q-3		**Attempt all questions**	
	a.	Find the Fourier series of the periodic function with period 2π defined by $f(x) = \begin{cases} -1, & -\pi < x < 0 \\ 1, & 0 < x < \pi \end{cases}$.	**(05)**
	b.	Find the Fourier cosine integral representation of $f(x) = \begin{cases} \sin x; & 0 \le x \le \pi \\ 0; & x > \pi \end{cases}$.	**(05)**
	c.	Find the inverse Laplace transform of $\frac{e^{-as}}{s^2+1}, a > 0$.	**(04)**

SECTION – II

Q-4		**Attempt the Following questions**	**(07)**
	a.	Find the Laplace transform of $f(t) = t$.	
	b.	Write error function.	
	c.	Define: Unit Step function.	
	d.	Find $Z(-1)$.	
	e.	If $L[f(t)] = \bar{f}(s)$, then $L^{-1}\left[\frac{\bar{f}(s)}{s}\right] = \int_0^t f(t)\, dt$. Determine whether statement is True or False.	

f. If $L[f(t)] = \bar{f}(s)$, then $L[t^n f(t)] = (-1)^n \frac{d^n}{ds^n}\bar{f}(s)$. Determine whether statement is True or False.

g. Z-transform of unit impulse sequence is $\frac{z}{z-1}$. Determine whether statement is True or False.

Q-5 **Attempt all questions** **(14)**

a. Obtain the inverse Laplace transform of $\frac{4s^2-3s+5}{(s+1)(s^2-2s+2)}$. **(07)**

b. State and prove First shifting theorem. **(04)**

c. Prove that $Z\{f_{n+k}\} = z^k\left[F(z) - f_0 - \frac{f_1}{z} - \cdots - \frac{f_{k-1}}{z^{k-1}}\right]$ **(03)**

OR

Q-5 **Attempt all questions**

a. State and prove Convolution theorem. Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s}{(s+a)(s^2+1)}\right\}$. **(07)**

b. Find the inverse Z- transforms of $\frac{8z^2}{(2z-1)(4z-1)}$. **(04)**

c. If $u(x,t)$ is a function of two variables x and t, then prove that $L\left[\frac{\partial u}{\partial t}, s\right] = sU(x,s) - u(x,0)$. **(03)**

Q-6 **Attempt all questions** **(14)**

a. Using the Laplace transform, solve $y'' + 3y' + 2y = t\,e^{-t}$ given $y(0) = 1$ and $y'(0) = 0$. **(07)**

b. Prove that $Z(\cos n\theta) = \frac{z\,(z-\cos\theta)}{z^2-2z\cos\theta+1}$ and $Z(\sin n\theta) = \frac{z\sin n\theta}{z^2-2z\cos\theta+1}$, if $|z| > 1$. **(04)**

c. Show that a set of functions $\left\{\sin\frac{n\pi x}{c}, n = 1,2,3,\ldots\right\}$ is orthogonal on $(0,c)$ and find corresponding orthonormal set. **(03)**

OR

Q-6 **Attempt all Questions**

a. Using the Laplace transform, solve the following IBVP **(07)**

PDE: $u_{tt} = u_{xx}\,,\ 0 < x < 1, t > 0$

BCs: $u(0,t) = u(1,t) = 0,\ t > 0$

ICs: $u(x,0) = \sin\pi x\,, u_t(x,0) = -\sin\pi x\,, 0 < x < 1.$

b. Prove that $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n}\left(e^{-x^2}\right)$. **(04)**

c. Find the Z-transform of $\frac{1}{n}$. **(03)**